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Money and Credit with Asymmetric Information

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This paper studies the use of cash and credit for making transactions when there is asymmetric information in credit markets. For relatively low inflation rates, equilibria are of the pooling variety and low-credit-risk consumers signal their riskiness to lenders only indirectly by establishing a good track record in credit markets. For higher inflation rates there may exist a separating equilibrium in which low-credit-risk consumers directly signal their type to lenders by specializing in cash early in life and specializing in credit later in life. The greater the degree of adverse selection in credit markets, the wider the range of inflation rates for which a separating equilibrium exists. Credit usage is increasing in the inflation rate, but greater adverse selection may increase or decrease credit usage depending on the parameterization. *Journal of Economic Literature* Classification Number: E44.

1. INTRODUCTION

One of the main issues that plagues monetary economics has been explaining patterns of use of the various transactions media (*e.g.*, cash, credit, barter) and predicting how those patterns might be affected by factors such as alternative regulations and monetary policies. Overwhelmingly, market transactions in industrialized economies are carried out by cash and centralized forms of credit, yet frictionless neoclassical models predict that barter, or perhaps bilateral credit, are sufficient for facilitating most transactions. The reason that frictionless neoclassical models make these predictions is that there is simply no need for cash or for intermediated credit when agents meet repeatedly in centralized markets.

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Some recent strides have narrowed significantly the gap between observed transactions media and the predictions of theoretical models. One important stride is understanding how introducing certain types of frictions into models can explain why fiat money might have value in exchange. For instance, Lucas (1980) and Townsend (1980, 1987) describe spatial models in which cash can overcome an absence of a double coincidence of wants between paired traders, P. Diamond (1984) highlights the value of cash when agents must search for trading partners, and King and Plosser (1986) stress the value of cash in aligning agents' income and consumption streams in an economy with private information. These particular studies do not, however, incorporate alternative transactions media, and thus it is not possible in these models to study substitution between exchange media.

Others have studied less restrictive models that do allow for alternative transactions media and thus enable one to study substitution between media of exchange. Freeman (1989), Kiyotaki and Wright (1989), and Engineer and Bernhardt (1991) study models in which agents may use cash or they may barter. A second group of studies, which include Bernhardt (1989), Schreft (1992), Prescott (1987), and Ireland (1992a,b), considers models which generate a transactions demand for both cash and pairwise debt or trade credit. This second group of studies is particularly appealing in light of the observed fact that many individuals do have some margin of substitution between cash and credit for undertaking transactions. Bernhardt (1989) and Ireland (1992b) show that cash is used for transactions between agents that meet infrequently and credit is used between those that meet repeatedly. Relatedly, Schreft (1992) (see also Ireland, 1992a) shows that cash is useful for transactions in distant markets and credit is useful in local markets. Prescott (1987) uses a similar model to show that cash will be used for small purchases and credit for large purchases when there are fixed costs of using credit.

This paper also studies a model in which agents may use either cash or credit to transact in goods markets. The distinguishing feature of the environment is that individual consumers have better information about the riskiness of their income streams than do others and this introduces an adverse selection problem into credit markets. Using cash for transactions is costly because idle cash balances are eroded by inflation. The cost to an individual of using credit depends on how much information lenders have about his riskiness. In contrast, the cost of using credit in the studies by Schreft (1992), Ireland (1992a), and Prescott (1987) is attributable to (exogenous) transaction or contracting costs, and in Bernhardt (1989) credit costs arise because agents meet randomly and thus the date of loan repayment is stochastic.

After describing the model in the next section, Section 3 studies a benchmark case in which lenders know each borrower's riskiness. Here there is no adverse selection in credit markets and in this case the model has the property that consumers with certain income streams—*low-risk consumers*—minimize transactions costs by using exclusively credit for transactions. Consumers with variable income streams-*high-risk consumers*—face a higher cost of using credit because they might have insufficient income to repay creditors and these losses are recouped from other high-risk borrowers that do have sufficient income to repay creditors. In comparison, Section 4 studies the case in which lenders cannot observe directly each borrower's riskiness, and thus low-risk consumers bear an adverse selection cost in a pooling equilibrium and a signaling cost in a separating equilibrium. The relative size of these costs of using credit and the cost of using cash are the key determinants of the equilibrium use of cash and credit in the economy.

Conditional on the proportion of high-risk borrowers in credit markets, it is shown that if the inflation rate is sufficiently high then low-risk consumers signal their type directly to lenders through their cash and credit choices. These choices involve using more cash and less credit early in life than would be the case if riskiness was directly observed by creditors. For lower rates of inflation, the equilibrium is of the pooling variety in which all consumers whose type is unknown to lenders use identical amounts of credit. If this level of credit is low enough that even agents with relatively variable income streams are certain to be able to repay it, then no information about differences in riskiness is revealed over time in credit markets. However, if this level of credit is large enough that consumers with more variable incomes might be unable to repay creditors, then consumers are slowly sorted by riskiness over time.

The greater the proportion of high-risk borrowers, the greater the degree of adverse selection in credit markets. It is shown that economy-wide credit usage is increasing in the inflation rate but may increase or decrease with greater adverse selection depending on the parameterization. Further, greater adverse selection will widen the range of inflation rates for which there exists a separating equilibrium. However, even for very high degrees of adverse selection, there may still be a possibly large range of (relatively low) inflation rates for which low-risk borrowers do not signal directly their type in credit markets. Finally, because the benefit to low-risk consumers of signaling their type is increasing in both the degree of adverse selection and the inflation rate, for very small degrees of adverse selection direct signaling in credit markets occurs only for very high inflation rates.

The next section of the paper describes the model used to examine the implications of private information in credit markets for cash and credit use. The framework is a Townsend-type (Townsend, 1980, 1987) spatial model with finite-lived generations of consumers. Barter is infeasible because of an absence of a double coincidence of wants between traders, and bilateral credit is precluded by limited intertemporal tie-ins between consumers. A role therefore exists for firms who specialize in the provision of credit to consumers. This credit competes with cash as an instrument to facilitate transactions and is universally acceptable as a means of payment because creditors can completely diversify lending risk. The credit provided by these firms is similar to credit cards, a line of credit from a bank, or generic bank loans. In addition to diversification of risk, consistent with the arguments of Black (1970) and Fama (1980), an important activity of creditors is that they specialize in *evaluating* the riskiness of individual borrowers.

After describing the model in Section 2, Section 3 discusses the benchmark case where each consumer's income riskiness is known by creditors. Section 4 examines the economy with imperfect information and presents the principal findings of the paper. Section 5 reviews some key findings.

2. THE MODEL

2a. *The Environment*

The model is a variant of Lucas' version of the Cass–Yaari (1966) circle as discussed by Townsend (1980) and is modified to include credit-issuing firms.[1] Each period $t = 1, 2, \ldots$, a continuum of three-period-lived consumers is born. A consumer's age is common knowledge. It will become clear that agents will seek to obtain credit only in their first and second periods of life because income is received only at the end of each of these two periods. Two periods of credit market activity are the minimum required for reputation effects to arise in credit markets. A longer lifespan would not change the basic results of the paper. Consumers have linear preferences defined over consumption of the F (perishable) goods in the economy and they discount the future by a common factor $\beta \in (0, 1]$. As will be formalized below, the role of many goods is to generate an absence of a double coincidence of wants between pairs of consumers and thus a role for transactions media.

There are two types of consumers and a consumer's type is his private information. Consumers may differ only by the variability of their income streams. Specifically, type r consumers have uncertain income each period and type s consumers have certain income. Fraction γ_r of each generation of consumers are of type r and the parameter γ_r is common knowledge. To ensure that type is not revealed trivially after one period, it is assumed that a consumer's *ex post* income is observable only to himself.

[1] See Marquis and Reffett (1992) for a similar modification to a spatial model.

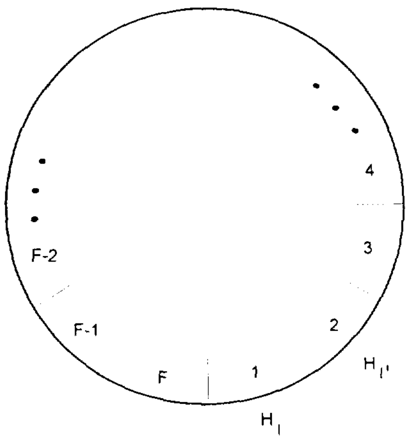

FIG. 1. Spatial locations are given on the inside of the circle. Credit-issuing firms reside permanently at each spatial location. Households' current positions are given on the outside of the circle. Households want to consume the good in the hands of the households one location counterclockwise. Endowments are received at clockwise adjacent trading posts represented by spikes.

Private information about type and about *ex post* income introduces an important friction between borrowers and lenders.

Consumers are born at one of F locations on a Cass–Yaari circle—see Fig. 1. Fraction $1/F$ of each new generation is born at each location with proportion γ_r being of type r. Denote by H_l the collection of consumers at location l in period t; there are F of these groups of consumers, one per location. A consumer born at location l in period t remains with the group H_l throughout his lifetime, but groups change locations each period in such a manner that any two groups, H_l and $H_{l'}$, are neighbors at most once. Following Townsend (1980), the trading opportunities (discussed below) each period will be limited to include trade only between households at adjacent locations. Thus, this structure on relocation between periods motivates limited intertemporal tie-ins between individual consumers at different locations. This friction will be central to the type of credit institutions that can arise in the economy.

A consumer's income is in the form of an endowment of location-specific goods. Formally, denote by $d_n^{jf}(t)$ the endowment of good f for a consumer of type j and of age n and currently located at f. In each period t, endowments are

$$d_n^{sf}(t) = a, \qquad n = 1,2, f = 1, \ldots, F, \tag{1}$$

$$d_n^{rf}(t) = a + \varepsilon(t), \qquad n = 1,2, f = 1, \ldots, F, \tag{2}$$

where

$$\varepsilon(t) = \begin{cases} -L \text{ with probability } \pi \\ U \text{ with probability } 1 - \pi, \end{cases}$$

and

$$-\pi L + (1 - \pi)U = 0, \qquad \text{with } L, U > 0.$$

It is also assumed that $a \geq \max\{L, U\}$ and that the random variable $\varepsilon(t)$ is independent and identically distributed (iid) across the population of type r consumers and over time. The implication of the iid assumption on endowments is that the per capita supply of each of the F goods is the same and averaging across agents at a given location yields an average endowment of $a(N - 1)/N$.

Consumers' tastes are such that those located at f obtain utility *only* from consumption of good $(f + 1)$. A household consists of a pair of agents and this allows it to perform simultaneous spatially separated transactions. At each date t, each member of a household located at f is capable of moving one-half the distance to each of the two adjacent locations, $(f - 1)$ and $(f + 1)$. Note that barter is precluded in this economy because there is an absence of double coincidence of wants between households at the same location *and* between households at adjacent locations. Further, bilateral credit cannot facilitate goods exchanges because households trade with each other at most once.

The above discussion implies that credit must derive from a third party. For type r consumers to be risky borrowers it is assumed that all consumers observe their current endowment after credit has been arranged and after consumption goods have been purchased. Formally, suppose that consumers at location f must travel halfway to location $(f - 1)$—called "market $(f - 1, f)$"—to observe (privately) and receive the current endowment. The member of the household whose duty this is sells the endowment to households that have traveled from location $(f - 1)$ to market $(f - 1, f)$. Concurrently, the second member of the household purchases good $f + 1$ at market $(f, f + 1)$. Note that the "shopper" may not know current income (at least if the household is of type r) while shopping.[2] Figure 2 summarizes the sequence of events each period.

Credit is issued by a large number of profit-maximizing firms called "creditors" stationed permanently at each location. Households obtain credit (*e.g.*, a bank loan, a line of credit, or perhaps a credit card) prior to

[2] A scheme in which households at a given location agree at the start of each period to pool endowments *ex post* and redistribute the mean amount to all consumers and thereby eliminate heterogeneous credit risks is not incentive compatible because endowments are private information.

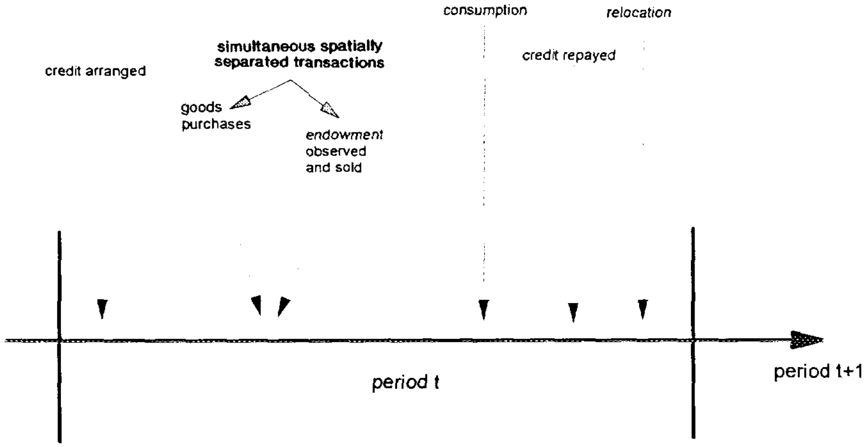

FIG. 2. Sequence of events.

their departure to goods markets and thus prior to their learning of current income. Creditors at different locations communicate only indirectly through their mutual access to information about consumers' credit histories maintained by a centralized "credit bureau." At a minimum, a credit history consists of information about the amounts of credit granted to any consumer and whether or not the consumer defaulted. Richer information structures that include previous interest rates and other data would not change the results. What is crucial is that a consumer's income history is not in creditors' information sets. Credit agreements can be made contingent only on the information available to the creditor, which includes the information detailed above as well as observables such as age. Because there is only limited communication between creditors at different locations and because consumers reside at a location at most once, credit agreements are limited to one-period contracts. Thus, any credit extended to a consumer in the current period must be repaid at the end of the period, prior to his relocation.

It is well known that if a borrower's income is private information then the optimal contract is a standard debt contract provided either that lenders can verify a borrower's income at a cost (Gale and Hellwig, 1985; Townsend, 1979) or there are non-pecuniary penalties for defaulting (D. Diamond, 1984). Following D. Diamond (1984), debt contracts are motivated in this paper because of the presence of default penalties.[3] Diamond

[3] I assume that creditors offer debt contracts taking default penalties as given since if creditors are permitted to choose the level of default penalty for any debt contract offered, it might be possible to sort consumers and overcome any adverse selection. Motivation for this assumption is that my interest here is with the signaling role of cash/credit choices by consumers rather than in designing optimal screening mechanisms for creditors.

suggests these might be interpreted to include factors such as time spent in bankruptcy proceedings, costly explaining of insufficient funds, search costs of a known defaulter, and the consumer's loss of reputation in bankruptcy.[4]

Cash competes with credit for use in facilitating transactions. At $t = 1$ agents are endowed equally with $M(0)$ units of fiat money. There exists a government whose sole activity is the printing of fiat money, the proceeds of which are used to finance government consumption. Government consumption is spread evenly over the F goods in each period and this yields no utility to consumers or creditors. If $g(t)$ denotes consumption of each of the F goods in a symmetric allocation, $p(t)$ denotes the nominal price level at any location f, and $M(t)$ the per capita money stock, then

$$g(t) = \frac{M(t) - M(t-1)}{p(t)} \geq 0. \tag{3}$$

Define μ to be the gross growth rate in the money stock. It is assumed that at $t = 0$ the government chooses $\mu \geq 1$, which, given the price level, implies from (3) a sequence $\{g(t)\}$. This formulation means that government expenditures are determined endogenously.

Figures 1 and 2 jointly characterize the structure of the model. The first event each period is that households obtain credit from creditors. The shopper from each household then travels to the goods market one-half the distance to the adjacent location counterclockwise and purchases goods using credit and any cash on hand that was carried over from the previous period. Concurrently, the worker proceeds one-half the distance to the adjacent location clockwise, learns of the current endowment, and then sells it, accepting either credit or cash. The two members of a household then return to their initial location and consumption occurs. Finally, payments to creditors are made, cash is accumulated for any excess of goods sales over payments to creditors, and then agents move to a new location where they arrive at the start of the next period.

Since there is a continuum of consumers at each location, so long as a positive fraction obtains credit from each creditor, then creditors can diversify all idiosyncratic risk. It follows that there is no risk of a creditor being unable to honor credit that it has granted. Hence, credit issued by all creditors is perfectly substitutable, and therefore consumers are indif-

[4] For this model non-pecuniary penalties are more convenient than Townsend's (1979) costly state verification approach or Diamond's (1989) liquidation technology. This is because in the model of this paper payments are made to creditors after consumption occurs and thus liquidation costs or verification costs cannot be in units of goods. Nonetheless, it is possible in this model to interpret the default penalties as simply state verification costs that are passed on to the defaulters.

ferent to which creditor or consumer they do business with. Since agents may receive cash as well as claims on creditors when they sell their current endowments, consumers repay creditors with these claims and obtain cash for any excess. Finally, one can imagine period-by-period centralized clearing and settlement operations for these claims, although the symmetry in this environment makes the interlocation clearing function trivial.

Next, I formalize decision problems and define an equilibrium for this economy. Since the decision problems for consumers of a given type are symmetric across locations, in what follows I focus on representative consumers of each type.

2b. *Decision Problems and Definition of Equilibrium*

The consumer–creditor relationship is modeled as a signaling game. A consumer approaches a creditor with a request for a particular amount of credit and the creditor responds with an interest rate. The consumer then either accepts or rejects the offer. If rejected, the consumer may seek out another creditor and the sequence is repeated. Denote by $D^j_n(t)$ the credit level requested by a consumer of type j and age n in period t and let $r^j_n(t)$ be the associated *interest-rate factor* (*i.e.*, one plus the interest rate) offered. That is, the consumer owes $r^j_n(t)D^j_n(t)$ at the end of period t.

The analysis is restricted to symmetric and stationary allocations in which, for all type j consumers, $D^j_n(t) = D^j_n$ and $r^j_n(t) = r^j_n$, and also $p(t + 1)/p(t) = \mu$. The per capita real money stock is therefore a constant, $\tilde{M} = M(t)/p(t)$. Further, if $m^j_n(t)$ denotes *end-of-period* nominal cash balances (*i.e.*, after payments to creditors) of the agent, then let $\tilde{m}^j_n = m^j_n(t)/p(t)$ denote end-of-period real cash balances. *Beginning-of-period* real balances at age $(n + 1)$ are therefore $\tilde{m}^j_n\mu^{-1}$. This shows clearly that using cash for transactions is costly because the real value of idle cash balances is eroded by inflation. Finally, C^j_n denotes the amount of consumption of the agent in his nth period of life.

As discussed above, credit contracts are incentive compatible because of the presence of non-pecuniary default penalties. D. Diamond (1984) shows in a static model without money that the minimum penalty that induces incentive compatibility of debt contracts is max[$r^j_nD^j_n$-payment, 0], where "payment" denotes the payment from the consumer to the creditor. The intuition is simple: If agents have linear consumption, then a non-pecuniary penalty equal to the difference between the amount owed creditors and the payment to creditors is the minimum penalty required to induce either full repayment or whatever is possible. In the model here, intentional default results in carrying additional cash balances into the next period. Since real balances are taxed by inflation and consumption is discounted by a factor β, the Diamond penalty is therefore multiplied by

(β/μ), giving a penalty function $q^j_n \equiv (\beta/\mu)$ max$[r^j_n D^j_n$-payment, 0]. With this penalty function, consumers will always repay principal plus interest or else whatever they are capable of paying: payment = min$\{r^j_n D^j_n, d^j_n + (\tilde{m}_{n-1}\mu^{-1} + D^j_n - C^j_n)\}$. Note that because households have income only in their first two periods of life, the presence of this default penalty deters them from seeking credit in their last period of life.

This schedule for default penalties is rather delicate and for this reason it should be noted that any fixed default penalty of magnitude $L/(1 - \pi)$ or greater is sufficient to induce incentive compatibility of debt contracts. I use the Diamond penalty because it simplifies some of the algebra below, but otherwise it is not crucial for the qualitative results.

Define a *credit history* h_n of a consumer of age n, $n = 1, 2$, as the collection $h = \{D_{n-1}, I_{n-1}\}$, where I_{n-1} is an indicator function that is equal to 1 if the agent defaulted at age $n - 1$ and 0 otherwise. Clearly, if the agent is of age 1 then the agent has no credit history. Thus, one can write $h_1 = \{\ \}$ and $h_2 = \{D_1, I_1\}$. Denote the collection of all possible one-period credit histories as $\mathcal{H}_2$. Second, define the *income history* y_n of the agent as $y_n = \{d_{n-1}\}$, where again $y_1 = \{\ \}$. Note that an agent's income history is his private information. The collection of all possible one-period income histories is denoted $\mathcal{Y}_2$.

The decision problem of consumers is as follows. An age 1 consumer of type j chooses a contingent strategy for cash and credit use, $\{D_n(h_n, y_n, j), \tilde{m}_n(h_n, y_n, j)\}^2_{n=1}$, in order to maximize expected discounted lifetime utility subject to budget constraints, and takes as given interest rates and penalty functions for defaulting. Formally,

$$\text{(Program P1)} \qquad \max_{\{D_n(h_n, y_n, j),\, \tilde{m}_n(h_n, y_n, j)\}^2_{n=1}} E_1 \sum_{n=1}^{3} \beta^{n-1}(C^j_n - q^j_n)$$

subject to

$$C^j_n \leq \tilde{m}^j_{n-1}\mu^{-1} + D^j_n, \tag{4}$$

$$\tilde{m}^j_n = \max\{d^j_n + (\tilde{m}^j_{n-1}\mu^{-1} + D^j_n - C^j_n) - r^j_n D^j_n, 0\}, \tag{5}$$

$$q^j_n = (\beta/\mu)\max\{r^j_n D^j_n - d^j_n - (\tilde{m}^j_{n-1}\mu^{-1} + D^j_n - C^j_n), 0\}. \tag{6}$$

Here, E_1 denotes the expectation conditional on consumer i's information set, which includes all public information as well as the agent's type and income history. Inequality (4) states that consumption must be financed either by cash or credit, and (5) states that end-of-period cash balances are equal to current income plus beginning-of-period cash balances and credit less consumption and payments to creditors. Equation (5) defines the initial condition that consumers are born with no cash, $\tilde{m}_{n-1} = 0$ for $n = 1$. The final constraint, (6), is the Diamond penalty function as discussed above. Using (4), third-period consumption is $C_3^j = \tilde{m}_2^j \mu^{-1}$, since agents have no income in their last period of life.

Next, consider the decision problems of creditors. A creditor receives a request from an agent of age n for credit amount D_n. The creditor evaluates the riskiness of the applicant based on $\{h_n, D_n\}$ and offers an interest-rate factor $r(h_n, D_n)$ that maximizes expected profit.[5] Formally

(Program P2)

$$\max_{r(h_n, D_n)} E(\min\{r(h_n, D_n)D_n, d_n + \tilde{m}_{n-1}\mu^{-1} + D_n - C_n\} - D_n),$$

where E denotes the creditor's expectation conditional on his information set, which includes h_n and D_n, although not the consumer's type. As formalized below, because creditors act competitively this implies that expected profits are, in equilibrium, equal to zero.

DEFINITION. A (symmetric, stationary, pure-strategy) perfect Bayesian–Nash equilibrium for the incomplete-information economy is a set of non-negative contingent credit levels and end-of-period real cash balances $\{D_n^*(h_n, y_n, j), \tilde{m}_n^*(h_n, y_n, j)\}$ for $j \in \{s, r\}$, $n = 1, 2$, $h_2 \in \mathcal{H}_2$, and $y_2 \in \mathcal{Y}_2$, a set of one-period interest rates $\{r^*(h_n, D_n)\}$ for $n = 1,2$ and all $h_2 \in \mathcal{H}_2$, and a set of posterior beliefs $\{\eta(h_n, D_n) = \text{prob}(\text{type } r | \{D_n, h_n\})\}$ for $n = 1, 2$ and all $h_2 \in \mathcal{H}_2$, such that:

(i) The collection $\{r^*(h_n, D_n)\}$ solves Program P2 conditional on beliefs $\{\eta(h_n, D_n)\}$, and beliefs satisfy Bayes' rule on the equilibrium path.

(ii) For each type $j \in \{s, r\}$, the plan $\{D_n^*(h_n, y_n, j), \tilde{m}_n^*(h_n, y_n, j)\}$ solves Program P1 conditional on interest rates $\{r^*(h_n, D_n)\}$.

(iii) Markets clear—average end-of-period real balances equal $\bar{M}$, and each credit contract gives a competitive expected gross return to lenders of unity (*i.e.*, the value function implied by Program P2 is equal to zero for each contract.)

Since the strategy space is continuous there will in general exist a multiplicity of allocations that satisfy the above definition. I restrict attention to the set of *Pareto-undominated allocations* that satisfy this definition. This restriction yields a unique equilibrium for any parameterization. Denote such allocations as PBNEs. Before determining the set of PBNEs for the incomplete-information economy it is useful to consider first the simpler complete-information equilibrium. This is the subject of the next section.

[5] Note that beginning-of-period money holdings are not a credible signal of type (*e.g.*, they can be concealed).

3. COMPLETE INFORMATION IN CREDIT MARKETS

Suppose that creditors can directly observe a consumer's type, but not his realized income. In conjunction with the penalty for defaulting, the latter assumption ensures that standard debt contracts are optimal. Consider first type s consumers. Since these agents are certain not to default on any credit level up to their current income a, then the competitive interest-rate factor will be unity on credit requests up to a. Alternatively, because income is received at the end of the period, should an agent wish to finance consumption purchases with cash, each unit of the current endowment effectively yields only β/μ units of current utility because of the one-period lag in the spendability of cash receipts from endowment sales. Since $\mu \geq 1$ and $\beta \leq 1$, then $(\beta/\mu) \leq 1$. It follows that for type s consumers cash is more costly than credit for financing consumption. Thus, type s consumers' optimal strategy is to finance all consumption purchases with credit: $D_n^s = a$, $n = 1, 2$.[6] The competitive interest-rate factor is $r_n^s = 1$, $n = 1, 2$.

The optimal strategy for type r consumers is more complicated because these consumers have a random income stream and thus the interest-rate factor offered to type r consumers may be greater than unity in order to compensate creditors for the losses suffered on the subset of consumers that have low current income and thus default. The cost of credit for these consumers will clearly depend on the amount of credit requested since that will affect the amount by which agents might default.

Type r consumers have no cash at the start of their first period of life but have real cash balances of $\tilde{m}_1\mu^{-1}$ at the start of their second period of life. In each of their first two periods of life they must decide how much credit to request, and in their second period they must also decide whether to spend all cash on hand. Because it is costly to hold idle cash balances and because the default penalty (6) does not punish agents so severely that they would want to carry cash simply to guard against default, then agents will spend in each period any cash on hand at the start of the period. With regard to the amount of credit to acquire, since there are no strategic features to the decision problem when there is complete information, then the credit decision is age independent and thus agents will choose the same credit level at ages 1 and 2.

Type r consumers have three options for financing consumption purchases: (i) use only cash, (ii) use only credit, or (iii) use a combination of cash and credit. Using exclusively cash for transactions delivers an expected lifetime utility of

[6] When $\beta/\mu = 1$, type s agents are indifferent between holding cash and credit in equilibrium.

$$EU^r(cash) = (\beta + \beta^2)a/\mu. \tag{7}$$

Using exclusively credit for transactions implies a lifetime credit sequence of $D_1^r = D_2^r = a$. The competitive interest-rate factor is denoted r^r, which is given by (8):[7]

$$r^r = [(1 - \pi)a + \pi L]/(1 - \pi)a. \tag{8}$$

Note that principal plus interest costs in this case are $r^r D_n^r = [(1 - \pi)a + \pi L]/(1 - \pi) = a + U$. That is, consumers who do not default (*i.e.*, fraction $1 - \pi$ of type r consumers) repay their entire income, $a + U$, whereas those that do default (fraction π) repay $a - L$ (and incur non-pecuniary penalties of $(\beta/\mu)(L + U)$). Expected lifetime utility for a type r consumer can be written as follows:

$$EU^r(a, r^r) = (1 + \beta)[a - (\beta/\mu)\pi L/(1 - \pi)]. \tag{9}$$

The final option available to type r consumers is to use a combination of cash and credit. Suppose that the credit level chosen is $D' < a$ in each of the first two periods of life. The competitive interest-rate factor on D' is

$$r' = [D' - \pi(a - L)]/(1 - \pi)D'. \tag{10}$$

Because consumers are choosing a credit level which is less than expected wealth, they expect to hold end-of-period real balances of $(a - D')$. One can therefore write expected lifetime utility in this case as follows:

$$\begin{aligned} EU^r(D') = (1 + \beta)[D' - (\beta/\mu)[\pi/(1 - \pi)][D' - a + L] \\ + \beta\mu^{-1}(a - D')]. \end{aligned} \tag{11}$$

First consider the decision to use either exclusively credit for transactions or else a combination of cash and credit. From (9) and (11), we have $EU^r(D') > EU^r(a, r^r)$ if the following condition is satisfied:

$$\mu/\beta < 1/(1 - \pi). \tag{12}$$

Before discussing this condition it is helpful to state the following result which sharpens the possible credit levels that type r consumers will choose in any full-information equilibrium. (The proofs of all propositions are contained in the Appendix.)

[7] Due to the costs of default, consumers (of either type) would never, in equilibrium, request a credit level greater than their mean income, a.

PROPOSITION 1. *With full information, there are only two possible equilibrium levels of credit used by type r consumers. These credit levels are a and $a - L$.*

The implication of this result is that if type r consumers use both cash and credit for transactions, then they will use a credit amount of $D' = a - L$ (with associated competitive interest-rate factor $r = 1$) and expected end-of-period real cash balances of L. Inequality (12) is therefore a necessary and sufficient condition for using credit amount $a - L$ and expected end-of-period real cash balances of L to provide lower transactions costs (and thus higher expected utility) than using exclusively credit for transactions. The converse of this statement is also true. The intuition behind (12) is that, if μ/β is relatively small, then the cost of using cash for transactions is relatively low. This puts upward pressure on the demand for cash and downward pressure on the demand for credit. There is a second effect working in the same direction: A low μ/β implies that the non-pecuniary default penalty is relatively large, which puts downward pressure on credit demand and upward pressure on the demand for cash. Exactly how small μ/β must be for these effects to be operative, however, depends on the parameter π. If π is large (small) then inequality (12) is more (less) likely to be satisfied.

The final option is to use exclusively cash for transactions. Note that using a credit amount of $a - L$ and expected end-of-period real cash balances of L weakly dominates using only cash for transactions. This is so because the competitive interest-rate factor on credit amount $a - L$ must be unity, which cannot exceed the per unit (gross) cost of using cash, μ/β.

To summarize, with full information type s consumers use exclusively credit to make transactions. Cash is a more costly transactions media for these agents because they receive credit at an interest rate that reflects their true riskiness and because idle cash balances are subject to the inflation tax. The cost of using credit is higher for consumers with more variable income streams and they will use more (less) credit if the cost of using cash is relatively high (low). For $\mu/\beta \in (1, 1/(1 - \pi))$, type r consumers use the riskless amount of credit $a - L$ and have expected end-of-period real balances of L. For higher costs of using cash, $\mu/\beta \in (1/(1 - \pi), \infty)$, type r consumers use an amount of credit equal to their mean income each period and use no cash for transactions. Note that in this second case cash is not used by any consumers for transactions and thus it is valueless (and thus government expenditures are zero).

Next, I examine the implications of creditors being unable to observe directly a consumer's income riskiness.

4. INCOMPLETE INFORMATION IN CREDIT MARKETS

The possible equilibria when there is incomplete information are described in Propositions 3 and 4. Proposition 2 establishes that a separating equilibrium does not exist for a range of low inflation rates, regardless of the degree of adverse selection, γ_r.

PROPOSITION 2. *For $\mu/\beta \in (1, 1/(1-\pi))$ and $\gamma_r \in (0, 1)$ there does not exist a separating PBNE. That is, if type s consumers choose a credit level D_n^s, then type r consumers whose type is not revealed by their credit histories will also choose credit level $D_n^r = D_n^s$, for $n \in \{1, 2\}$.*

This result rules out the possibility of low-risk consumers signaling their type directly in credit markets through their cash/credit choices when the inflation rate is relatively low. The reasoning behind this result is as follows. For $\mu/\beta < 1/(1-\pi)$, recall from Section 3 that when riskiness is known to creditors type r consumers choose credit level $a - L$ and type s consumers choose credit level a. In contrast, if riskiness is not known to lenders then note that high-risk consumers always have an incentive to masquerade as low-risk consumers because they would receive a greater amount of credit with lower interest rates. Moreover, this implies that they would have lower cash holdings, which further reduces transactions costs.

The implication of Proposition 2 is that for a range of low rates of inflation the only possible equilibria are of the pooling variety. The possibilities are described in the next result.

PROPOSITION 3. *Define δ as*

$$\delta \equiv \left[\frac{[1-\beta^2(1-\pi)^2](1-\gamma_r\pi)}{[1-\beta(1-\pi)][(1-\gamma_r\pi)^2 + \beta(1-\pi)(1-\gamma_r) + \beta(1-\pi)^3\gamma_r]}\right]. \tag{13}$$

δ is a monotone increasing, concave function of $\gamma_r \in (0, 1)$ with range $(1, 1/(1-\pi))$. For a given degree of adverse selection, γ_r, there are two possible equilibria depending on the rate of inflation:

(i) For $\mu/\beta \in (1, \delta)$, the unique PBNE outcome is where all consumers use credit amount $a - L$ in each of the first two periods of life, the equilibrium interest rate is $r^* = 1$, and expected end-of-period real cash balances of all consumers are L.

(ii) For $\mu/\beta \in (\delta, 1/(1-\pi))$ the unique PBNE outcome is as follows. All type s consumers and type r consumers that have not previously defaulted use credit amount a/r_n^p in period $n = 1, 2$, of life and equilibrium interest-rate factors are $r_1^p = a/(a - \gamma_r\pi L)$ and $r_2^p = a(1 - \gamma_r\pi)/[a(1 -$

$\gamma_r\pi)-\gamma_r\pi L(1 - \pi)]$. *These consumers have zero expected end-of-period real cash balances. Type r consumers that have previously defaulted use credit amount $a - L$, the competitive interest-rate factor is $r^* = 1$, and expected end-of-period real cash balances are L.*

Proposition 3 establishes a unique equilibrium for $\mu/\beta \in (1, \delta)$ in which all consumers use the riskless amount of credit, $a - L$. Since type s consumers have end-of-period real cash balances of L, then from (4) one can calculate type s consumers' consumption pattern: $C_1^s = a - L$, $C_2^s = a - L(1 - \mu^{-1})$, and $C_3^s = L\mu^{-1}$. *Type r* consumers also have first-period consumption of $a - L$ but their second-period consumption will depend on their first-period income. Fraction π of type r consumers have zero end-of-period cash balances and fraction $1 - \pi$ have end-of-period cash balances of $L + U = L/(1 - \pi)$. Thus, using (4), second-period consumption of type r agents is $a - L$ for those that had zero end-of-period cash balances in the previous period and is $a - L[1 - 1/\mu(1-\pi)]$ for the others. Third-period consumption of type r consumers is financed by cash carried into the period, which is equal to either zero or $L\mu^{-1}$.

For $\mu/\beta \in (\delta, 1/(1 - \pi))$, the equilibrium is more complicated because the level of credit that agents pool on is greater than the riskless amount. This means that fraction π of type r consumers have income $a - L$ and thus will default in their first period. Since default signals type, these agents' optimal cash/credit choice in their second period of life is given by the complete information results of Section 3: they consume $a - L$ in their second period of life financed by credit and have expected consumption of $L\mu^{-1}$ in their third period of life financed by cash. Those type r agents that do not default in their first period have income $a + U$, and thus will carry real cash balances of $U\mu^{-1}$ into their second period. Consumption of these agents in their second period is the sum of current credit a/r_2^p and beginning-of-period real cash balances, $U\mu^{-1}$. Finally all type s consumers will consume a/r_n^p in period $n = 1, 2$, and nothing in their third period of life. Note that the smaller amount of adverse selection in credit markets in consumers' second period of life lowers the interest rate: $r_1^p > r_2^p$. If consumers had longer lifespans than assumed here then the prediction is that interest rates decrease with the number of periods of credit market activity. Indeed, with infinite horizons consumers would, in the limit, be sorted perfectly by riskiness.

The fact that consumers' credit choices for $\mu/\beta < 1/(1 - \pi)$ are one of two possible amounts parallels Proposition 1. In particular, with imperfect information about borrower type, consumers choose a credit amount of $a - L$, a/r_n^p, or a. The intuition for using a credit amount of a/r_n^p stems from the fact that this credit level yields maximum benefit (in the sense of lower costs of credit) to type r consumers of "pooling" with type s consumers in credit markets. In contrast, a credit level of $a - L$ yields minimum cost to type s consumers from pooling with type r consumers.[8]

The parameter δ is clearly important for credit usage. Because δ is an increasing function of γ_r, the greater the degree of adverse selection in credit markets, the wider the range of monetary growth rates for which consumers use the smaller amount of credit $a - L$ and relatively more cash. Indeed, since $\delta \rightarrow 1/(1 - \pi)$ as $\gamma_r \rightarrow 1$, then if the number of low-risk consumers is insignificant we have that the riskless credit level $a - L$ is the unique equilibrium credit choice for $\mu/\beta < 1/(1 - \pi)$. Further, $\delta \rightarrow 1$ as $\gamma_r \rightarrow 0$ so that all consumers choose a credit level of a when the degree of adverse selection is insignificant. Of course, in light of the full-information solution, these findings are precisely what one would expect.

Comparing Proposition 3 with the complete-information analysis of Section 3, it is apparent that type s consumers' cash/credit decisions are distorted when there is private information about income riskiness. In general, adverse selection in credit markets causes type s consumers to use more cash and less credit. Further, their consumption is lower either because credit costs are higher (due to adverse selection) or because they use cash, the real value of which is eroded by inflation. On the other hand, type r consumers' cash/credit choices (and consumption) are unaffected by the presence of private information for $\mu/\beta \in (1, \delta)$, but they use more credit and less cash (and their consumption is higher) for $\mu/\beta \in (\delta, 1/(1 - \pi))$. Put differently, for $\mu/\beta \in (1, 1/(1 - \pi))$, the presence of private information transfers wealth from low-risk consumers to the government (through seigniorage) for $\mu/\beta < \delta$ and to high-risk consumers (through interest rates) for $1/(1 - \pi) > \mu/\beta > \delta$.

Next, I present the final result which characterizes the possible equilibria for higher rates of inflation: $\mu/\beta > 1/(1 - \pi)$. In contrast to Proposition 2, it is shown in Proposition 4 that low-risk consumers might signal directly their type to creditors through their cash/credit choices in their first period of life. Before presenting the formal result, it is helpful to consider Fig. 3. The lower portion—below $1/(1 - \pi)$ on the vertical axis—summarizes the equilibrium credit choices as described in Proposition 3 above. Shown in the upper portion of Fig. 3 are the following: (i) a horizontal line at $(1 + \beta\pi(1 - \pi))/(1 - \pi)$, (ii) a curve denoted by α, and (iii) a curve denoted by χ. Proposition 4 establishes that direct signaling in credit markets occurs if the cost of using money for transactions, μ/β, is above the horizontal line (i) *and* above the lower of the curves (ii) and (iii). The formal result is:

[8] That I restrict attention to Pareto-undominated PBNE is crucial for this result.

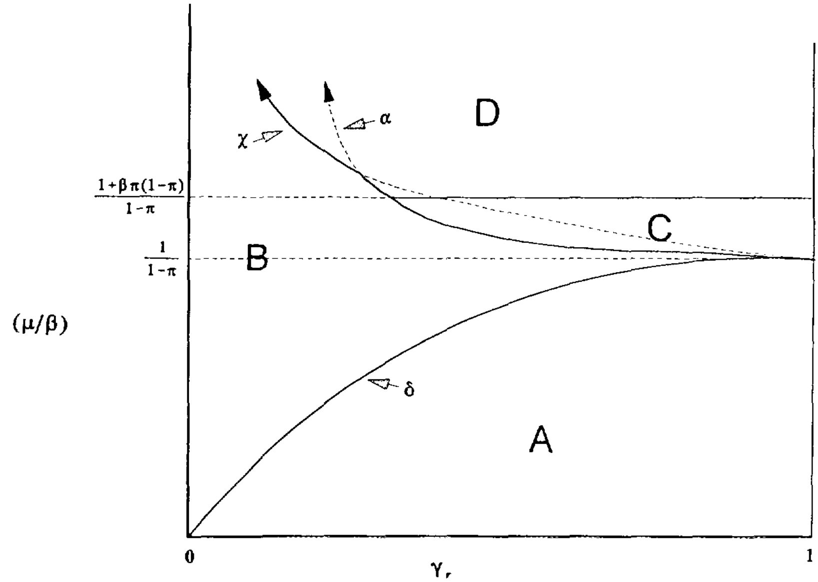

FIG. 3. Equilibrium credit usage. (Region A) All consumers use credit level $a - L$. (Region B) All consumers with good track records use credit level (a/r_n^p). Consumers with poor track records use credit level $a - L$. (Region C) All consumers use credit level a. (Region D) (Separating equilibrium) Type s consumers use credit level $\overline{D}_1 < a$ in the first period and credit level a in the second period. Type r consumers use credit level a.

PROPOSITION 4. *Let χ and α denote two monotone decreasing functions of γ_r with the following limiting properties*: $\alpha \to 1/(1-\pi)$ for $\gamma_r \to 1$, *and* χ, $\alpha \to \infty$ *for* $\gamma_r \to 0$. *The possible equilibria for* $\mu/\beta > 1/(1-\pi)$ *are as follows*:

(i) For $\mu/\beta > \max\{(1+\beta\pi(1-\pi))/(1-\pi), \min\{\alpha, \chi\}\}$ *(region D in Fig.* 3), *there exists a separating equilibrium with the following properties. Type s consumers choose an age* 1 *credit of* $\overline{D}_1^s < a$, *an age* 2 *credit of* a, *and end-of-period real cash balances of* $(a - \overline{D}_1^s)$ *at age* 1 *and zero cash balances in other periods, and the equilibrium interest-rate factor is* $r^* =$ 1. *Type r consumers choose credit amount a in each of the first two periods of life and zero cash balances, and the equilibrium interest-rate factor is* $r^r = [(1-\pi)a + \pi L]/(1-\pi)a$.

(*ii*) *For* $\mu/\beta \in (1/(1-\pi), \min\{\alpha, \chi\})$ *(the portion of region B above* $1/(1-\pi)$ *in Fig.* 3), *the unique PBNE is as described in Proposition* 3(*ii*). *For all other values of* $\mu/\beta > 1/(1-\pi)$ *(region C in Fig.* 3), *there is a unique PBNE in which all consumers choose credit amount a, the interest-rate factor is* r^r, *and cash is not used by any consumer.*

From Section 3, if lenders know a borrower's type, then for $\mu/\beta > 1/(1 - \pi)$ a type r consumer's optimal lifetime strategy is to choose credit level a (with associated competitive interest-rate factor r^r) and use no cash for transactions. Denote this strategy as (a, r^r). When type is not known by lenders, Proposition 4 states that it might be possible for type s consumers to choose an age 1 credit level $\bar{D}_1^s < a$ such that type r consumers have no incentive to deviate from (a, r^r). If such a $\bar{D}_1^s$ exists, then this is an informative (and costly) signal of type and creditors will extend credit level a to these consumers at the riskless rate of interest in their second period of life. Consumption of type s agents reflects these cash/credit dynamics: $C_1^s = \bar{D}_1^s$, $C_2^s = a + (a - \bar{D}_1^s)\mu^{-1}$, and $C_3^s = 0$.

Why might high-risk consumers choose not to mimic low-risk consumers and thereby reap the benefits of pooling with them? The answer to this question is that the costs of holding the additional cash balances associated with choosing a credit level less than a, in conjunction with the probability of defaulting on this credit level (and one's type being revealed) and thus being denied the favorable rates of interest in their second period of life, can make mimicking sub-optimal.[9] In contrast, low-risk consumers are certain not to default either on the credit level $\bar{D}_1^s$ or on the credit level a in their second period of life. Moreover, low-risk consumers may have much to gain from establishing a *reputation*. In Diamond's (1989, p. 829) words: "Reputation effects on decisions arise when an agent adjusts his or her behavior to influence data others use in learning about him." This is precisely the objective of type s consumers in choosing $\bar{D}_1^s$—to acquire a reputation.

In Proposition 2 it was established that low-risk consumers could not signal their type directly when $\mu/\beta < 1/(1 - \pi)$. This result is attributable to the fact that for $\mu/\beta < 1/(1 - \pi)$ type r consumers would *necessarily* benefit from mimicking the behavior of type s consumers since they would receive a *larger* amount of credit at lower interest costs and incur *lower* costs of using cash. In contrast, for $\mu/\beta > 1/(1 - \pi)$, choosing the same strategy as type s consumers would mean receiving a *lower* credit level (than the complete-information optimum) and thus bearing *larger* costs of using cash. It follows that it is not necessarily optimal for type r consumers to mimic type s consumers, and thus a separating equilibrium may exist for $\mu/\beta > 1/(1 - \pi)$.

Loosely put, for it to be feasible for low-risk borrowers to signal their type directly to lenders through credit usage, the inflation rate must be sufficiently large to deter high-risk consumers from wanting to choose the same credit amounts. The lower bound on the inflation rate that is re-

[9] If $\bar{D}_1^s > a - L$ then the probability of defaulting is π. If $\bar{D}_1^s = a - L$, then there is zero probability of defaulting.

quired is formalized in Proposition 4 and shown as the solid-line boundary of region D in Fig. 3 (a formal treatment is in the Appendix). As the figure shows, the greater the degree of adverse selection in credit markets, the wider the range of inflation rates consistent with a separating equilibrium. Indeed, as $\gamma_r \rightarrow 1$, a separating equilibrium exists for all $\mu/\beta > (1 + \beta\pi(1 - \pi))/(1 - \pi)$. The intuition here is that more adverse selection increases the cost of credit to low-risk consumers in any pooling equilibrium and this increases the benefits to signaling their type directly in credit markets. In turn, this translates into a larger range of monetary growth rates for which there exists a separating equilibrium. This range is limited from below by the number $(1 + \beta\pi(1 - \pi))/(1 - \pi)$ because for values of μ/β less than this type r consumers will always find it beneficial to mimic type s consumers. In contrast, for very small degrees of adverse selection, the number $\min\{\alpha, \chi\}$ is very large so that the lower bound on the range of inflation rates for which a separating equilibrium exists is very large: a separating equilibrium exists only for very high inflation rates. This is because the benefits to type s consumers of direct signaling are low for small degrees of adverse selection.

Not made explicit in Fig. 3 or Proposition 4 is the level of credit, $\overline{D}_1^s$, chosen by low-risk consumers in a separating equilibrium. In any separating equilibrium, type s consumers will select the largest value for $\overline{D}_1^s$ such that a type r consumer's welfare is only marginally lower from pursuing the same strategy as type s consumers. This is so because lower values of $\overline{D}_1^s$ are more costly signals to creditors. In general, this means that $\overline{D}_1^s$ is increasing μ/β (the costs of using cash). Indeed, for $\mu/\beta \rightarrow \infty$, $\overline{D}_1^s \rightarrow a$ in any separating equilibrium. The fact that relatively low inflation rates rule out a separating equilibrium stems from the fact that it becomes impossible to choose $\overline{D}_1^s$ such that type s consumers are better off under this strategy *and* type r agents do not wish to mimic type s agents. Finally, recognize from Proposition 4 that cash is always valued for $\mu/\beta > 1/(1 - \pi)$ because it is used by at least some agents regardless of the equilibrium, although this usage becomes arbitrarily small for very large monetary growth rates.

Comparing Proposition 4 with the results discussed in Section 3 reveals the effect of private information about income riskiness for cash/credit use. In a separating equilibrium, low-risk consumers use less credit and more cash early in life when type is private information, but type r consumers use precisely the same amounts of cash and credit when income riskiness is private information and when it is not. However, when the equilibrium is of the pooling variety—in Fig. 3, the portion of region B above $1/(1 - \pi)$ as well as region C—the low-risk and high-risk consumers use either the same amount of credit when riskiness is private information (region C) or else they all use less credit (the portion of region

B above $1/(1 - \pi)$). Here too we observe that private information in credit markets reduces the consumption of low-risk consumers to the benefit of either the government (through seigniorage) or high-risk consumers (through pooling in credit markets).

Figure 3 reveals that the likelihood of a separating equilibrium is increasing in both the inflation rate and the degree of adverse selection. This makes sense since the benefits to low-risk consumers of lenders knowing their type are increasing in these parameters. Further, for a given γ_r, the economy-wide amount of credit used for transactions is weakly increasing in the inflation rate. Finally, note that the amount of credit used for transactions may be increasing or decreasing in the degree of adverse selection, depending on the parameterization. For example, if $1/(1 - \pi) < \mu/\beta < (1 + \beta\pi(1 - \pi))/(1 - \pi)$, then a higher degree of adverse selection can move the equilibrium from region B to region C, which means an increase in credit usage. In contrast, for $\mu/\beta < 1/(1 - \pi)$, higher degrees of adverse selection lead to lower credit usage.

Consider the effects of the parameters on the various equilibria when there is imperfect information about income riskiness. Consider first the parameter π which captures the riskiness of type r consumers since the variance of their income is increasing in π. Referring to Fig. 3, π enlarges the area below the point $1/(1 - \pi)$, but it can also be shown that an increase in π increases δ as well as the number $\max\{(1 + \beta\pi(1 - \pi))/(1 - \pi), \min\{\alpha, \chi\}\}$. A key implication here is that the range of monetary growth rates for which there exists a separating equilibrium is decreasing in the variability of type r consumers' income. This seemingly counter-intuitive result stems from the fact that the more variable the incomes of type r consumers, the greater the incentive they have to masquerade as type s consumers. A further implication of a larger value of π is that since δ increases in π the range of monetary growth rates for which all agents wish to use the riskless credit level $a - L$ and relatively more cash is wider.

The effect of β is similar. Specifically, the larger is β, the lower is δ and the larger is $\max\{(1 + \beta\pi(1 - \pi))/(1 - \pi), \min\{\alpha, \chi\}\}$. Therefore, the range of monetary growth rates for which there exists a separating equilibrium is decreasing in β. The range of monetary growth rates for which agents pool on credit level a/r_n^p is increasing in β, and the range of monetary growth rates for which they choose the riskless credit level $a - L$ is decreasing in β.

5. CONCLUSION

The objective of this paper was to study the implications for cash and credit use of private information about income riskiness in credit markets.

When borrowers' credit risks are not known by lenders, the relative costs of using cash and credit for transactions are determined endogenously by the mix of credit risks in the credit market. The presence of high-risk borrowers increases the costs of transacting for lower-risk borrowers, either because all agents are offered similar credit contracts (a pooling equilibrium) or because these agents must deviate from their full-information optimum cash/credit choices in order to communicate their true credit risk to lenders (a separating equilibrium).

This paper has shown that, for a given mix of high-risk and low-risk borrowers, there is a lower bound on the inflation rate such that for inflation rates above this bound, low-credit-risk consumers choose short-run sub-optimal amounts of cash and credit for transactions in order to signal their riskiness to lenders. This signaling behavior allows these consumers to use credit later in life with the costs of credit appropriate for their riskiness. Further, the greater the proportion of high-risk borrowers, the lower is this lower bound and thus the wider the range of inflation rates for which a separating equilibrium exists. For inflation rates below this lower bound, all consumers with a good track record in credit markets use the same amount of credit, although generally different amounts of cash because of different income distributions. Consumers may or may not be sorted by riskiness over time in a pooling equilibrium, depending on whether or not the amount of credit used for transactions is large enough that high-risk consumers might default.

APPENDIX

Proof of Proposition 1. In any equilibrium in which all borrowers' types are known to creditors, type r consumers will choose a credit level of $D' < a$ only if (12) holds. Suppose that (12) holds. Noting that the competitive interest-rate factor for credit level $a - L$ must be unity since no type r consumer will default, it follows that the expected lifetime utility of choosing credit level $a - L$ and expected (end-of-period) real balances of L is $(1 + \beta)[a - L(1 - \beta\mu^{-1})]$. This expected utility exceeds that given by (11) if $\mu/\beta < 1/(1 - \pi)$, which is true by assumption. Thus, type r consumers will use credit level $a - L$ in any complete-information equilibrium in which $\mu/\beta < 1/(1 - \pi)$. ∎

Strategies

To establish Propositions 2–4 it will be useful to determine first the expected utilities of consumers under various *conjectured strategies*. I first consider possible pooling strategies and then I study strategies where type r and type s consumers choose different credit amounts beginning in their first period of life.

(*A*) *Pooling strategies*. Consider strategies in which all consumers in their first period of life choose the same amount of credit for transactions. It will be shown below that, if it is optimal for a consumer to choose some *pooling* strategy at age 1, then it is optimal to choose that strategy at age 2. Hence, it is w.l.o.g. to consider age 1 consumers. The possible pooling strategies are: (i) no credit, (ii) no cash, (iii) credit level $a - L$, and (iv) credit level a/r_n^p at age n, for $n = 1, 2$. That attention can be restricted to these pooling strategies is discussed in Section 4. Specifically, restricting attention to Pareto-undominated equilibria there is only one credit level in $(a - L, a)$ which may be a pooling equilibrium.[10] Denote the competitive interest rate associated with this credit level r_n^p; this credit level is then a/r_n^p.

First, conjecture a pooling equilibrium in which all consumers choose (a, r^r), with r^r given by (8). The interest-rate factor r^r is independent of the mix of consumers, so long as a positive fraction are of type r.[11] Note that, since the interest rate is independent of age, if consumers then find it optimal to choose this strategy at age 1 then they find it optimal to choose it at age 2. Type s consumers will default on interest payments and incur non-pecuniary penalties of $\beta\mu^{-1}\pi L/(1 - \pi)$. Also, fraction π of type r consumers will default on both the interest payment and portion L of the principal. Fraction $1 - \pi$ of type r consumers will repay both interest and principal equal to $a + \pi L/(1 - \pi)$. This suggests the following expression for welfare in this conjectured pooling equilibrium:

$$EU^j(a, r^r) = (1 + \beta)[a - (\beta/\mu)\pi L/(1 - \pi)], \quad j = s, r. \tag{A1}$$

Consider next a strategy of choosing credit level $a - L$. Since all consumers can repay the principal with certainty, the competitive interest rate must be unity. Type s consumers will, with certainty, carry real cash balances of L between periods. Fraction $1 - \pi$ of type r consumers carry cash balances of U while fraction π carry no cash. Hence, type r consumers *expect* to carry cash balances of L. Thus, welfare in this case is

$$EU^j(a - L) = (1 + \beta)[a - L(1 - \beta\mu^{-1})], \qquad j = s, r. \tag{A2}$$

[10] The proof of this closely parallels Proposition 1 and is therefore omitted.

[11] Formally, consider the interest rate charged on a credit request of a for agents who have previously defaulted. By the arbitrage condition in the credit market, creditors must receive an expected gross return of 1 on credit granted to previous defaulters. This implies that the following condition must be satisfied:

$$[(1 - \gamma_r)/(1 - \gamma_r + \gamma_r(1 - \pi)^{n-1})]\, a + [(1 - \pi)^{n-1}\gamma_r/(1 - \gamma_r + \gamma_r(1 - \pi)^{n-1})] [\pi(a - L) + (1 - \pi)\, ra] = a.$$

Solving for r yields $r = r^r$. Hence, consumers who have previously defaulted and those that have not defaulted receive the same interest rate.

Since there is no age or time dependence, if consumers find it optimal to choose this strategy at age 1 then it is the optimal choice at age 2.

When consumers use only cash for consumption purchases, their expected lifetime utility is $(\beta + \beta^2)a/\mu$. For all $\mu/\beta > 1$, note that (A2) exceeds this level of expected utility. This is therefore a dominated strategy and will not be considered further.

The final conjectured pooling equilibrium is where all consumers choose credit level a/r_n^p. Note that fraction π of type r consumers will have defaulted by age 2, and thus the competitive interest-rate factor r_2^p will be lower than r_1^p. These interest rates are shown in Proposition 3. Since $r_2^p < r_1^p$, if consumers then find it optimal to choose credit level a/r_1^p at age 1, then they find it optimal at age 2.[12] Since type s consumers will not use cash for transactions in this case, we can write their *ex ante* welfare as

$$EU^s(a/r_n^p) = a/r_1^p + \beta a/r_2^p. \tag{A3}$$

In determining the expected lifetime utility for type r consumers one must account for the possibility that they might default and their type therefore revealed. One can capture this as

$$EU^r(a/r_n^p) = [a/r_1^p + \beta(1 - \pi)a/r_2^p] + \beta\pi[\text{BAS}], \tag{A4}$$

where BAS denotes "best available strategy," conditional on one's type being known to creditors. Of course, BAS can be found from the complete-information results reported in Section 3.

(B) Separating strategies. Next consider strategies by type s consumers which might upset a conjectured pooling equilibrium. Suppose first that the candidate pooling equilibrium is where all consumers choose credit level a at interest-rate factor r^r. Suppose that type s consumers "upset" this conjectured equilibrium by choosing a *first-period* credit level of $\overline{D}_1^s < a$. Suppose further that type r consumers have no incentive to deviate from the initial conjectured strategy. If this were the case, then it must also be true that type s consumers choose $\overline{D}_2^s = a$ with competitive interest-rate factor $r^* = 1$, since creditors observe consumers' credit histories and thus Bayes' Rule implies that creditors' beliefs will be that only type s consumers choose this strategy. Moreover, the competitive interest-rate factor for $\overline{D}_1^s$ must also be $r^* = 1$ if this is indeed an equilibrium.

[12] Note that it can never be the case that $a/r_n^p < a - L$, implying that $r_n^p = 1$. This is so because r_n^p is bounded above such that it can never attain $a/(a - L)$.

The expected utility of a type s consumer under this conjectured separating strategy is

$$EU^s(\bar{D}_1^s; a) = [\bar{D}_1^s + (a - \bar{D}_1^s)\beta\mu^{-1}] + \beta a. \quad \text{(A5)}$$

The expected utility of the *marginal* type r consumer from choosing the same strategy as type s consumers is

$$EU^r(\bar{D}_1^s; a) = [\bar{D}_1^r - \pi(\bar{D}_1^r - a + L)\beta\mu^{-1} + (1 - \pi)(a + U - \bar{D}_1^r)\beta\mu^{-1}] + \beta[(1 - \pi)^J a] + \beta[1-(1 - \pi)^J][a - \pi L\beta\mu^{-1}/(1 - \pi)], \quad \text{(A6)}$$

where J is an indicator function such that $J = 1$ if $\bar{D}_1^r = \bar{D}_1^s > a - L$ and $J = 0$ otherwise. The first term denotes the expected utility under strategy $\bar{D}_1^s$ (taking into account the probability of default and incurring the nonpecuniary default penalty), the second term the expected utility of credit level a with unit interest-rate factor (also taking the probability of default into account), and the third term the BAS if the consumer defaults and his type is revealed.

A second case is a conjectured pooling equilibrium in which all consumers choose credit level $a - L$. Proposition 2 establishes that it is not possible for type s agents to destroy this conjectured equilibrium by choosing a different strategy. I defer proof of that result until I discuss separating strategies that might upset the final possible conjectured pooling equilibrium.

The final instance in which a separating strategy could upset a conjectured pooling equilibrium is when consumers choose a credit level of a/r_n^p. Again, for the sub-case $\mu/\beta \in (1, 1/(1 - \pi))$ Proposition 2 establishes that this sub-case can be ruled out. The second sub-case is for $\mu/\beta \in (1/(1 - \pi), \infty)$. From Section 3, the BAS for type r consumers is (a, r^r) *if* type s consumers choose a separating strategy as described above *and* type r consumers also have no incentive to pursue this strategy. This is thus equivalent to the first case discussed above.

Proof of Proposition 2. Note from (A2) and (A1) that $EU^j(a - L) > EU^j(a, r^r)\ \forall\ \mu/\beta \in (1, 1/(1 - \pi))$ and $j = s, r$. If type r consumers are optimizing and choosing a credit level different from type s consumers, then it is clear from the analysis in Section 3 that they will choose a credit level of $a - L$. Thus, conjecture an equilibrium in which type r consumers choose a credit level of $a - L$ and type s consumers choose a credit level of $\bar{D}_n^{*s} > a - L$ with associated competitive interest-rate factor $r^* = 1$. Note that we can restrict attention to a single period since type r consumers have the option of choosing $a - L$ at $r^* = 1$ irrespective of their credit history. It is now shown that this cannot be a PBNE.

Note first that type r consumers receive time t expected utility in the proposed equilibrium of $EU^r(D^{*r} = a - L) = a - L(1 - \beta\mu^{-1})$. However, the time t expected utility from defecting and choosing $D^r = D^s > a - L$ is $EU^r(D^s) = D^r - \pi(D^r - a + L)\beta\mu^{-1} + (1 - \pi)(a + U - D^r)\beta\mu^{-1}$. Hence $EU^r(a - L) \geq EU^r(D^s)$ iff $a - L \geq D^s > a - L$, which is a contradiction. Thus, a separating PBNE does not exist for $\mu/\beta \in (1, 1/(1 - \pi))$. ■

Proof of Proposition 3. Proposition 2 ensures that one can restrict attention to pooling strategies for $\mu/\beta \in (1, 1/(1 - \pi))$. Consider first type r consumers. Recall that strategy $(a - L, r^* = 1)$ and expected cash balances of L weakly dominate using only cash for transactions for $\mu/\beta \geq 1$, which is true by assumption. Also, recall from Section 3 that $(a - L, r^* = 1)$ dominates (a, r^r) for $\mu/\beta < 1/(1 - \pi)$, which is also true by assumption. Hence, assuming that $(a/r_n^p, r_n^p)$ is an available strategy, we need only check the conditions under which $(a - L, r^* = 1)$ dominates $(a/r_n^p, r_n^p)$. Second, we must check the conditions under which $(a/r_n^p, r_n^p)$ is an available strategy—*i.e.*, the conditions under which type s consumers also choose this strategy.

It is easily shown from (A4) and (A2) that $EU^r(a/r_n^p) < [>]\ EU^r(a - L)$ iff $\mu/\beta < [>]\ \delta$, where δ is given by (13). Also from (A3) and (A2), $EU^s(a/r_n^p) < [>]\ EU^s(a - L)$, iff $\mu/\beta < [\geq]\ \delta^s$, where $\delta > \delta^s$ and

$$\delta^s \equiv \left[\frac{(1 - \beta^2)(1 - \gamma_r \pi)}{(1 - \beta)[(1 - \gamma_r \pi)^2 + \beta(1 - \gamma_r) + \beta(1 - \pi)\gamma_r (1 - \pi)^2]}\right]. \quad \text{(A7)}$$

Thus, if $\mu/\beta \in (1, \delta)$, then type r consumers' dominant pooling strategy is $(a - L, r^* = 1)$. The optimal *available* strategy of type s consumers is also $(a - L, r^* = 1)$. In equilibrium, creditors rationally believe that fraction γ_r of the pool of all borrowers are type r consumers.

Off-the-equilibrium-path beliefs of creditors that support this as an equilibrium is ζ, where $\zeta = P[\text{type } s|D_n \in (a - L, a]]$ such that the inequality

$$\sum_{n=1}^{2} \beta^{n-1}\{a - L + L\beta/\mu - D_n - I(a - r_n^1 D_n)\beta/\mu$$
$$+ (1 - I)(r_n^2 D_n - a)\beta/\mu\} > 0$$

holds, where I is an indicator function such that $I = 1$ if $a \geq r_n^* D_n$ and zero otherwise, and $r_n^1 = [D_n - (1 - \zeta)\pi(a - L)]/D_n[\zeta + (1 - \zeta)(1 - \pi)]$, $r_n^2 = [D_n - \zeta a - (1 - \zeta)\ \pi(a - L)/D_n(1 - \zeta)(1 - \pi)]$, which are simply the competitive interest-rate factors given beliefs ζ for $a \geq r_n^1 D_n$ and $a < r_n^1 D_n$, respectively. This inequality ensures that (A2) exceeds (A3) given beliefs ζ. This establishes part (*i*).

To establish part (*ii*), note that $EU^r(a/r_n^p) > EU^r(a - L)$ and $EU^s(a/r_n^p) > EU^s(a - L)$ for $\mu/\beta \in (\delta, 1/(1 - \pi))$. Thus, it follows that for $\mu/\beta \in (\delta, 1/(1 - \pi))$ type r consumers prefer $(a/r_n^p, r_n^p)$ and expected real cash balances πL to either $(a - L, r^* = 1)$ and expected cash balances L or (a, r^r) and zero cash balances. Moreover, type s consumers prefer $(a/r_n^p, r_n^p)$ and zero cash balances to either $(a - L, r^* = 1)$ and expected cash balances L or (a, r^r) and zero cash balances for all $\mu/\beta > \delta^s$. Thus, the unique equilibrium for all $\mu/\beta \in (\delta, 1/(1 - \pi))$ is as stated in part (*ii*) of Proposition 3. In equilibrium, creditors rationally believe that, of the pool of consumers of age n that have not previously defaulted, fraction $(1 - \gamma_r)/((1 - \gamma_r) + \gamma_r(1 - \pi)^{n-1})$ are type s consumers and that all consumers who have previously defaulted are type r consumers.

There is much freedom in choosing off-the-equilibrium-path beliefs that support this as an equilibrium. One set of beliefs is as follows. P[type $s| D_n \in (a - L, a/r_n^p]$; no previous defaults] $= (1 - \gamma_r)/((1 - \gamma_r) + \gamma_r (1 - \pi)^{n-1})$. The competitive interest-rate factor given these beliefs is

$$r = \{D_n[1 - \gamma_r + \gamma_r(1 - \pi)^{n-1}] \\ -\gamma_r\pi(1 - \pi)^{n-1}(a - L)\}/D_n[1 - \gamma_r + \gamma_r(1 - \pi)^n]$$

for any $D_n \in (a - L, a/r_n^p]$; P[type $s|D_n \in (a/r_n^p, \mathrm{a}]$; no previous defaults] $= (1 - \gamma_r)/((1 - \gamma_r) + \gamma_r(1 - \pi)^{n-1})$, and the competitive interest-rate factor given these beliefs is

$$r = \{D_n[1 - \gamma_r + \gamma_r(1 - \pi)^{n-1}] \\ - (1 - \gamma_r)a - \pi(1 - \pi)^{n-1}\gamma_r\}/\{D_n\gamma_r(1 - \pi)^n\}$$

for any $D_n \in (a/r_n^p, a]$; P[type $s|D_n \in (a - L, a]$; previous defaults] $= 0$ and best response $r = \{D_n - \pi(a - L)\}/\{(1 - \pi)D_n\}$ for $D_n \in (a - L, a]$. (Note that these beliefs satisfy Cho and Kreps' (1987) "intuitive criterion"—as do off-the-equilibrium-path beliefs in all of the equilibria. This establishes part (*ii*) of Proposition 3. ■

Proof of Proposition 4. A useful preliminary result is the following:

LEMMA 1.

$$EU^r(a/r_n^p) > EU^r(a, r^r), \quad \text{if, and only if, } \mu/\beta < \alpha, \tag{A8}$$

$$EU^r(\overline{D}_1^r; a) > EU^r(a, r^r), \quad \text{if, and only if, } \mu/\beta < \bar{\tau}, \tag{A9}$$

$$EU^s(\overline{D}_1^s; a) > EU^s(a, r^r), \quad \text{if, and only if, } \mu/\beta < \bar{\eta}, \tag{A10}$$

$$EU^s(\overline{D}_1^s; a) > EU^s(a/r_n^p), \quad \text{if, and only if, } \mu/\beta < \bar{\sigma}. \tag{A11}$$

Proof of Lemma 1. Lemma 1 makes four claims. Consider each in turn. To establish (A8), using (A4) and (A1) we have that the former exceeds the latter when

$$a/r_1^p + \beta(1-\pi)a/r_2^p + \beta\pi[a - (\beta/\mu)\pi L/(1-\pi)] \\ > (1+\beta)[a - (\beta/\mu)\pi L/(1-\pi)]. \tag{A12}$$

Collecting terms involving β/μ, cross multiplying, and simplifying, yields the equivalent condition that (A4) exceeds (A1) iff $\mu/\beta < \alpha$, where

$$\alpha \equiv \left[\frac{[1+\beta(1-\pi)][1-\gamma_r\pi]}{\gamma_r(1-\pi)[(1-\gamma_r\pi)+\beta(1-\pi)^2]}\right]. \tag{A13}$$

Equation (A9) can be established in a similar manner using (A6) and (A1). Specifically (A6) is easily shown to exceed (A1) if

$$\bar{D}_1^r + \beta a + (\beta/\mu)[(a - \bar{D}_1^r) - (\beta\pi L/(1-\pi))(1-(1-\pi)^J)] \\ > (1+\beta)a - (1+\beta)(\beta/\mu)\pi L/(1-\pi), \tag{A14}$$

where J is an indicator function such that $J = 1$ if $D_1^r > a - L$ and $J = 0$ otherwise. Solving for μ/β on the right side of the inequality, one can write (A14) alternatively as $\mu/\beta < \bar{\tau}$, where

$$\bar{\tau} \equiv 1 + \left[\frac{(\pi L/(1-\pi))(1+\beta(1-\pi)^J)}{a - \bar{D}_1^r}\right]. \tag{A15}$$

Equation (A10) can be established using (A5) and (A1). We have that the former exceeds the latter iff

$$\beta a + [\bar{D}_1^s + (a - \bar{D}_1^s)\beta\mu^{-1}] > (1+\beta)[a - \pi L\beta\mu^{-1}/(1-\pi)], \tag{A16}$$

where $J = 1$ if $\bar{D}_1^s > a - L$ and $J = 0$ otherwise. Isolating μ/β on the right side yields the equivalent condition that (A5) exceeds (A1) if $\mu/\beta < \bar{\eta}$, where

$$\bar{\eta} \equiv 1 + \left[\frac{(\pi L/(1-\pi))(1+\beta)}{a - \bar{D}_1^s}\right]. \tag{A17}$$

The final inequality is (A11). Using (A5) and (A3), and proceeding in a similar manner to the above derivations, (A5) exceeds (A3) if $\mu/\beta < \bar{\sigma}$, where

$$\bar{\sigma} \equiv \frac{(a - \bar{D}_1^s)}{(a - \bar{D}_1^s) - \gamma_r \pi L[1 + \beta(1 - \pi)/(1 - \gamma_r \pi)]} \tag{A18}$$

It is possible that (A18) is negative; in this case (A5) exceeds (A3) for all μ/β. This establishes Lemma 1. ■

Proposition 4, *Part* (*i*). It is helpful to distinguish two cases corresponding to μ/β greater than and less than α.

Case (*a*): $\mu/\beta > \alpha$. Note from Lemma 1 that a separating equilibrium exists in this case if type s consumers can choose $\bar{D}_1^s$ in such a manner that $\bar{\eta} \geq \mu/\beta \geq \bar{\tau}$. I begin by characterizing the properties of the functions $\bar{\eta}$ and $\bar{\tau}$. First, $\bar{\eta} > \bar{\tau}$ for all $\bar{D}_1^s \in (a - L, a)$. Second, $\bar{\eta} \to \infty$ and $\bar{\tau} \to \infty$ as $\bar{D}_1^s \to a$. Third, for $\bar{D}_1^s$ approaching $a - L$, $\bar{\eta}$ approaches $(1 + \beta\pi)/(1 - \pi)$ and $\bar{\tau}$ approaches $[1 + \beta\pi(1 - \pi)]/(1 - \pi)$. Clearly, both of these numbers are greater than $1/(1 - \pi)$ and $\bar{\eta}$ is greater than $\bar{\tau}$. The conclusion is that type s consumers can choose $\bar{D}_1^s$ to generate intervals $[\bar{\tau}, \bar{\eta}]$ than span $[(1 + \beta\pi(1 - \pi))/(1 - \pi), \infty)$. It follows that if $\alpha > (1 + \beta\pi(1 - \pi))/(1 - \pi)$ then there exists a separating equilibrium: $\bar{\tau} > \mu/\beta > \max\{\alpha, \bar{\eta}\}$. Since α is decreasing in γ_r with range $[1/(1 - \pi), \infty)$, then there exists a separating equilibrium for all $\mu/\beta > \max\{\alpha, (1 + \beta\pi(1 - \pi))/(1 - \pi)\}$. This subset of the parameter space is shown in Fig. 3 as that portion of region D above the curve α and above the horizontal line $(1 + \beta\pi(1 - \pi))/(1 - \pi)$. This establishes the first case.

Case (*b*): $\mu/\beta \in (1/(1 - \pi), \alpha)$. From Lemma 1, type r consumers prefer (a, r_n^p) to (a, r^r) in this case. Thus, as established in the proof of Proposition 4, part (ii), below, if a separating equilibrium does not exist in this case, the unique pooling equilibrium is $(a/r_n^p, r_n^p)$. It follows that $\bar{D}_1^s \in [a - L, a/r_1^p)$, for type r consumers would surely benefit from mimicking type s consumers if they chose a credit level exceeding a/r_1^p.

From Lemma 1, a separating equilibrium requires $\min\{\alpha, \bar{\sigma}\} > \mu/\beta > \bar{\tau}$. Since the function $\bar{\tau}$ has a lower bound of $[1 + \beta\pi(1 - \pi)]/(1 - \pi)$ for $\bar{D}_1^s \to a - L$, then it follows directly that there does not exist a separating equilibrium for $\mu/\beta \in (1/(1 - \pi), (1 + \beta\pi(1 - \pi))/(1 - \pi))$. This is the rectangle in Fig. 3 with vertical corners given by these endpoints, its height equal to $\beta\pi$ and its length equal to unity.

Consider the remaining parameter space $\mu/\beta \in ((1 + \beta\pi(1 - \pi))/(1 - \pi), \alpha)$, for $\alpha > (1 + \beta\pi(1 - \pi))/(1 - \pi)$. The issue is whether type s consumers can, for any given μ/β in this range and any given γ_r, choose $\bar{D}_1^s$ such that $\bar{\sigma} > \mu/\beta > \bar{\tau}$. It will be shown that this is possible only for a subset of this parameter space.

First consider small values of γ_r. Note that $\bar{\tau}$ is unaffected by γ_r. However, $\bar{\sigma} \to 1$ as $\gamma_r \to 0$, for all feasible $\bar{D}_1^s$, and this convergence is from above. Note also that for any positive but small value of γ_r, it is always possible to choose a $\bar{D}_1^s$ such that $\bar{\sigma} > \bar{\tau}$. To see this, note that $\bar{\sigma} \to \infty$ as $\bar{D}_1^s \to y$, where $y = a - \gamma_r \pi L(1 + \beta(1 - \pi)/(1 - \gamma_r \pi))$. However, $\bar{\tau}$ tends to infinity only for $\bar{D}_1^s \to a < y$. But as γ_r gets very small, the number y approaches a (from below). What this means is that it is always possible to choose $\bar{D}_1^s$ such that $\bar{\sigma} > \bar{\tau}$, but the space that is spanned by all possible intervals $[\bar{\tau}, \bar{\sigma}]$ is a subset of $((1 + \beta\pi(1 - \pi))/(1 - \pi), \alpha)$ which has a lower bound that is decreasing in γ_r. As γ_r tends to zero, this lower bound tends to infinity. This implies a curve—denoted χ—with the general features shown in Fig. 3. A formal derivation of the curve χ proceeds by choosing $\bar{D}_1^s \in (a - L, y]$ and selecting the value of γ_r such that $\bar{\sigma} = \bar{\tau}$. This establishes one point on the curve. Repeating this exercise traces out the curve completely.

To complete the characterization one needs to determine the behavior of the curve χ for larger values of γ_r. Of course, the arguments above determine the general properties here, but what is of more interest is the relation of the curve χ to the curve α. In general it is possible that the curve χ intersects the horizontal line $[1 + \beta\pi(1 - \pi)]/(1 - \pi)$ on either side of the curve α. The figure shows the case where χ intersects $[1 + \beta\pi(1 - \pi)]/(1 - \pi)$ to the right of α. It also shows that χ converges to $1/(1 - \pi)$; there are other possibilities here, but the dynamics of χ for $\chi > \alpha$ are not important for characterizing equilibria.

Collecting results, it has been established that there exists a separating equilibrium for all $\mu/\beta > \max\{(1 + \beta\pi(1 - \pi))/(1 - \pi), \min\{\alpha, \chi\}\}$. Moreover, one can further state that the optimal choice of $\bar{D}_1^s$ is the value that ensures $EU^r(\bar{D}_1^s; a) = EU^r(a, r^r)$, since this maximizes $EU^s(\bar{D}_1^s; a)$ while assuring $\mu/\beta > \max\{(1 + \beta\pi(1 - \pi))/(1 - \pi), \min\{\alpha, \chi\}\}$. In equilibrium, type s consumers choose $\bar{D}_1^s$ with interest-rate factor $r^* = 1$ and the credit level a with interest-rate factor $r^* = 1$ in their second period of life. Type r consumers choose credit level a with interest-rate factor r^r in the first two periods of their life. In equilibrium, creditors rationally believe that $P[\text{type } s | D_1 = \bar{D}_1] = 1$. This completes the proof of part (i).

Proposition 4, *Part* (ii). To establish the second part, note that, for $\mu/\beta > 1/(1 - \pi)$, a separating equilibrium does not exist for $\mu/\beta \in (1/(1 - \pi), \max\{(1 + \beta\pi(1 - \pi))/(1 - \pi), \min\{\alpha, \chi\}\})$. From Lemma 1, all consumers prefer (a, r^r) and no cash balances to $(a - L, r^* = 1)$ and expected cash balances L. Thus, we need only check the conditions under which consumers prefer $(a/r_n^p, r_n^p)$ to (a, r^r).

From Lemma 1 type r consumers prefer the former to the latter iff $\mu/\beta < \alpha$. Consider type s consumers. It is easily shown that (A3) exceeds

(A1) iff $\mu/\beta < \alpha^s$, where

$$\alpha^s \equiv \left[\frac{(1 - \gamma_r\pi)(1 - \beta^2)}{\gamma_r(1 - \pi)[1 - \gamma_r\pi + \beta](1 - \beta)}\right]. \tag{A19}$$

Comparing (A19) with (A9), it follows that $\alpha^s < \alpha$. Moreover, note that α is bounded below by $1/(1 - \pi)$. Thus, if $\mu/\beta \in (1/(1 - \pi), \min\{\alpha, \chi\})$ (the portion of region B above $1/(1 - \pi)$ in Fig. 3), then type r consumers prefer $(a/r_n^p, r_n^p)$ to (a, r^r). This need not be true for type s consumers since $\alpha > \alpha^s$. Nonetheless, the dominant strategy of type r consumers which have not previously defaulted is to choose $(a/r_n^p, r_n^p)$. This is, therefore, the unique pooling equilibrium outcome (supported by appropriate beliefs—see below) for $\mu/\beta \in (1/(1 - \pi), \min\{\alpha, \chi\})$. In this equilibrium, type s consumers hold no cash balances and type r consumers have expected cash balances of πL. Type r consumers who have previously defaulted choose (a, r^r) and no cash balances. The beliefs on the equilibrium path and off the equilibrium path that support this as an equilibrium are as given above in the Proof of Proposition 3, part (*ii*).

The second case is region C in Fig. 3. Key to region C is that $\mu/\beta > \alpha$, and thus the dominant strategy of all consumers is (a, r^r). It follows that this is the unique pooling equilibrium. Off the equilibrium path beliefs that support this as an equilibrium is a $\zeta = P(\text{type } s|D_n \in (a - L, a]]$ chosen to satisfy the conditions analogous to those stated in the proof of Proposition 3(*i*). This establishes part (*ii*) of Proposition 4. ■

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